Why Your Child Can't Understand Math

By Gerry Schnell

Introduction

A fierce war rages over control of your child's mathematics education. It is a cruel war, fought mostly on the sly, in which the only casualties are children. The war is between those who want your child to understand mathematics and those who just want her to learn it. (For convenience, I'm going to call "your child" Dana from now on). The stakes for Dana are enormous. Leaving aside personal benefits, which are substantial, a good math education is a vital key to entry to a good, or even not so good, university. Once there, it is a key to entry into the school of her choice within that university. And a good education is the key to a comfortable and productive life. Income distribution now is based largely on the level and quality of education, with highly educated people moving swiftly into well paying jobs and becoming part of the "cognitive elite." Others toil for a pittance and suffer the bumps and jolts that come with life in the slow lane.

Why is proficiency in mathematics required for college work? Certainly it is required if Dana wants to be an engineer or physicist or mathematician, but why is it required for a career in literature or philosophy or poetry? The answer is that proficiency in math is used as a filter to reduce the number of *qualified* applicants to many schools. State universities, for example, are required to accept all qualified resident applicants. In today's climate, where the importance of a college degree is clear to all, applications flood those universities. The response is to shrink the pool of qualified applicants by raising the entrance requirements. Math proficiency makes a great barrier, especially in view of the terrible job in math education being done in so many high schools and grade schools. And what if Dana has great mechanical aptitude and would make a fine engineer if she chose to become one, except for a distaste for math acquired in grade school? She can probably grit her teeth and make it into the university system, but she is likely to end up with a degree in history and a job in human resources, not engineering or science. In brief, she will find it harder to approach her potential in life without a good background in math.

The message of this booklet is that a young child cannot understand math, not even arithmetic, because her brain is not yet wired for the task. She can learn to *perform* arithmetic, all of it, but she can't understand it. Virtually no one in her age group can, not anywhere in the world. But there are those who believe she *can* understand arithmetic, that she can *explore* the subject and *discover* the underlying principles, and that she can come to *understand* it through that process. That is the reason for the war, and that is the reason Dana can be harmed by a system that commands her to do something she cannot do.

What is it about grade school arithmetic that is impossible for children to understand? Our base 10, Arabic numeral system of working with numbers certainly seems simple enough. It's not rocket science, after all. Or is it? We will see that the system is a very sophisticated method of handling numbers. It is the end product of thousands of years of thought by some very, very smart people. We also will see that a child's brain is psychologically immature. It is not wired to duplicate (rediscover) the work of mature scholars from the past. Someday, Dana may stand on the shoulders of those scholars and make contributions of her own, but first she must sit at their feet and become expert in *performing* their method. When all is said and done, it is impossible to measure a schoolchild's understanding of mathematics. But it is quite possible to measure performance, and that is exactly what college entrance boards do.

Before we visit the battlefields of this war for control of Dana's mathematical education, let's take a look at the complexity of the subject matter and the mental equipment she brings to the classroom.

Levels of mathematical sophistication — pre-modern

A. Hunter-Gatherers

There are a few hunter-gatherer societies still in existence, and we may assume that their level of mathematical development has remained unchanged throughout the millennia. Many of these cultures have no words for numbers higher than “two” or “three.” In these societies, numbers sometimes are attached to specific objects. Thus, there may be no generic term for, much less abstract concept of, the number "two." The “two” used to quantify one type of animal may differ from the “two” used to quantify another. There is a faint echo of this in modern English when we refer to a *brace* of pheasants or a *pair* of twins. The people are primitive (by our standards), but their brain structures are not. For example, there are no primitive languages. The languages of the most primitive societies are as grammatically complex and rhetorically supple as those of the most modern societies. It is not that primitive people lack the brain-power to comprehend more sophisticated mathematical structures, but rather that they lack the need for a higher concept of mathematics. In other words, they don’t have much to count. Australian Aborigine children who are exposed to “modern” educational structures perform as well, on average, as their nonnative counterparts.

B. Herding Societies

Herdsmen have numbers high enough to count the animals in their herds. There is no concept of numbers as abstract entities or of formal arithmetic operations. If four animals are to be sold to one buyer and a price per animal is agreed upon, each animal must be sold in a separate transaction to avoid confusion and mistrust. As with the hunter-gatherers, they have developed mathematical structures sufficient to meet their needs, and no more.

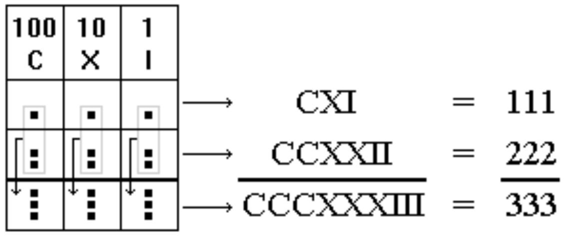
C. Ancient Civilizations

These societies had words and symbols sufficient to generate very large numbers, and, though they needed mechanical aids to perform calculations, they could do the arithmetic for any activity from state bookkeeping to huge construction projects to planning grain production for whole populations. The Roman Empire was one such civilization, and its method of using Roman numerals to record numbers, together with counting boards to perform the arithmetic, is the one that eventually was replaced by Arabic numerals and algorithms.

The Roman method, which did not use place value, expressed each number as the addition or subtraction of certain symbols, as with MM DC LXXX VIII, which we express as 2,688. It is called an *additive* system of notation as opposed to our *positional* system of notation. Like symbols were added to obtain a subtotal, and the subtotals were added to produce a total. I, X, C, and M were symbols for ascending powers of 10, with values of 1, 10, 100, and 1000, respectively, while V, L, and D were symbols for the intermediate values 5, 50, and 500. Larger numbers were formed by adding bars to the tops of letters. A single bar over “V,” for example, indicated a multiple of 1,000, for a final value of 5,000.

Additive systems, by their nature, do not accommodate paper and pencil work, and so counting boards and counters were required to perform arithmetic. An addition problem was expressed in Roman numerals, the operation was done on a counting board, and the result was written in Roman numerals. The counting board was a very simple adding machine that functioned as a conversion device between *additive* Roman numerals and *positional* place value, with numerals replaced by counters and the value of the counters determined by their place (the column they occupied) on the board.

The figure below illustrates the process of the Roman Counting Board. It is not a true reproduction.



The counters actually produced two values, additive and positional. Three individual counters *within a place* on the counting board were added to produce an additive value of 3. The particular place those three counters occupied determined whether they would be multiplied by 1, 10, 100, 1,000, or a higher power of ten. Thus, the three counters could have a positional value of 3, 30, 300, or 3,000, in the same way that the value of our "3" is determined by its place within a number. The difference is that our "3" is a symbol in which the addition (1+1+1) already has been done. It is a critical difference, as we shall see.

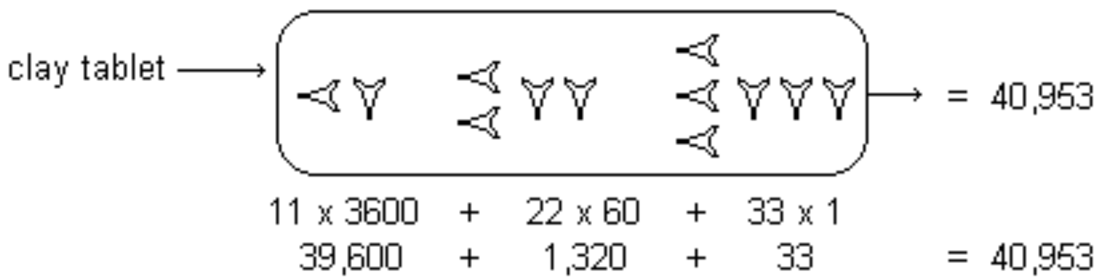
Although the use of Roman numerals and the counting board was a workable method of performing arithmetic and recording numbers, it still was primitive by comparison to our place value method of manipulating *numbers* instead of *objects*. Our modern system can deal with any number, no matter how large, using only ten symbols (0 - 9), but the Roman system, where ten "I"s made one "X," ten "X"s made one "C," and ten "C"s made one "M," required an infinity of symbols to handle an infinity of numbers.

That said, the Roman numeral – counting board system did indeed work. It met the needs of the Roman Empire, and no more. Almost anyone could learn to use it, and there was an added benefit in that it was difficult to alter Roman numerals on commercial documents. (It is comparatively easy to alter "333' by changing a "3" to an "8,' by adding a "0" at the end, or by adding a "9" at the beginning; that's why we express numbers on bank checks in both Arabic numerals and words). The system functioned well enough to remain in place until the greatly expanded commercial activities of the Industrial Revolution required a faster and more efficient method of dealing with numbers.

D. Babylonian base 60

Mesopotamia, one of the great centers of civilization of the ancient world, developed a unique base 60 numbering system using place value. (Base 60 means dealing with groups of 60 instead of groups of 10). A remnant of that system divides our modern days into seconds, minutes, and hours, with 60 seconds in a minute and 60 minutes, or 3600 (60 x 60) seconds, in an hour. For example, 11:22:33 A.M. means 11 hours, 22 minutes, and 33 seconds after midnight, a simplified expression of (11 x 3600) + (22 x 60) + (33 x 1) seconds after midnight (the total is 40,953, illustrated below). We know from archeological evidence that the system was employed in the days of King Hammurabi of Babylonia (c. 1700 BC), and the sophistication of the mathematical calculations indicates it was in use long before then. Base 60 used only two numerals (as opposed to our ten Arabic numerals or the Roman infinity of numerals), a vertical wedge shape that designated "1" and a horizontal wedge shape that designated "10." When combined, those symbols had both additive and positional values. Two horizontal wedges and two vertical wedges (10+10+1+1), for instance, had an *additive* value of 22. The positional value of 22 was determined by its place within a number. The figure below shows how the Babylonians would have recorded 11:22:33 (A.M.).

Babylonian place value notation with cuneiform numerals - these are not counters!



Base 60 had several advantages over base 10. Very large numbers could be generated and handled quite easily. The repetitive nature of the numbering system eliminated the need to assign differing symbols for the ascending powers of 60 (as opposed to the Roman method of using letters of the alphabet for the ascending powers of 10). Fractions were easier to handle because 60 has more factors than 10 (1, 2, 3, 4, 5, 6, 10, 15, 20, 30, and 60 all divide evenly into it). Perhaps the most important advantage was that base 60 dovetailed nicely with their 360-day calendar and 360 degree circle. The scientific communities of Greece, Rome, and eventually all Medieval Europe, used base 60 as a sophisticated alternative to the more common Roman (or Greek) numerals until it, too, was replaced by Arabic numerals and algorithms.

The Babylonian base 60 numbering system was the first recorded example of *written* place value and held splendid promise, but there were drawbacks. First, there was no zero. An empty *place* was held by an empty *space*. In the example "11 22 33," given above, if the middle *place* were empty (11 0 33), the number would be recorded "11   33." Carelessness with spacing caused uncertainty about the value of the number. Second, it required additive notation within each *place*, so that "59" would be written as 5 horizontal wedges and 9 vertical wedges, a clumsy system. (If Arabic numerals were restricted to two symbols, "5" and "1,' we would write "873" as "5111 511 111"). Therefore, evolution into a place value system similar to our own would have required 60 individual numerals (including zero), a disabling obstacle.

Babylonian astronomers and scholars who worked with base 60 were, to say the very least, extraordinary mathematicians who handled very complex problems with very simple tools. Counting boards, which were useful for base 10 arithmetic but not base 60, were not in play. Beyond that, high-end Babylonian math was far, far more complicated than simple arithmetic (although it's likely Babylonian merchants handled simple arithmetic in much the same way as the Romans). What the mathematicians did was solve complex problems once, then record the event on a clay tablet. Over time, the tablets evolved into formal tables that were stored in libraries and used by future generations to solve other, more complex problems. No one is sure how the original problems were solved.

Why were Babylonians forced to rely on large libraries of mathematical tables (inscribed on clay tablets) instead of working individual problems out using the equivalent of paper and pencil? One reason is that was no papyrus, much less paper. Clay was the available medium. Papyrus had been invented in Egypt, but didn't appear in Mesopotamia until much later. A more fundamental reason is that manipulating numbers on paper is not possible unless one already has memorized the requisite addition, subtraction, multiplication, and division facts. Excluding the zero, there are 9 working numbers in base 10. That means there are a total of 81 (9 x 9) addition facts, and 81 subtraction facts. Had base 60 been equipped with a full array of numerals (instead of only two), it would have been necessary to memorize 3,481 (59 x 59) addition facts, and an equal number of subtraction, multiplication, and division facts, for a grand total of 13,924 facts, an extremely daunting task, even for those mathematical heroes.

In any event, base 60 met many needs, and so it lived on and spread to the learned classes in Greece, then Rome, then, eventually, all of Europe. It also spread to the Indian subcontinent, and there it became involved in one of the most momentous events in the history of mankind.

The development of our modern numbering system

A. India and the birth of "Arabic" numerals

Like Greece and Rome, India retained the use of base 10 (with letters of the alphabet representing the ascending powers of 10) for trading purposes and for basic arithmetic. Both systems required the use of a counting board. Around 500 AD, a truly intelligent person got to thinking about how base 60 numeration worked so well using a limited number of numerals (two only, the vertical and horizontal wedges), and how convenient it was to have the value of these numerals determined by their position in a *number*, as the value of counters was determined by their position on a *board*. He must have pondered how nice it would be to have that same simplicity of repeating numerals in base 10.

Whatever his thought processes, this unique individual combined the principles of the counting board and base 60, and developed a system of repeating numerals for base 10. There were 9 numerals involved (1 - 9, zero had been developed by the Persians during their tenure in Mesopotamia around 300 BC, but the concept had not spread). Astoundingly, he devised a set of numerals in which the additive nature of the existing system was bypassed because the numerals were symbols for addition *that had already been done!* This insight occurred only once in the history of mankind, and so it seems a shame that the singular accomplishment of this unknown Indian genius has been given the misleading popular name *Arabic* numerals (the formal name is *Hindu Arabic* numerals, which seems small compensation). Roughly two hundred years later, Hindus incorporated zero into the system and almost all the components of our modern base 10, positional notation system of numbers were in place.

B. Islam constructs a cultural highway from India to Europe

In 711 AD, the recently established and rapidly expanding Islamic Empire launched successful military campaigns against both Europe and India, thereby connecting Europe to all the intellectual riches from Alexandrian Egypt to the Indus Plain. The cultural flow was one way, since what had been the Western Roman Empire was by then ruled by the children of barbarians. Islamic scholars acquired the Hindu base 10-place value system and, in ninth century Baghdad, a Persian mathematician named al-Khowarazmi published a book in Arabic about the new numbers and the rules (algorithms) for dealing with them. (*Algorithm* is a corruption of his name; the rules he passed along remain unchanged to this day).

Hindu Arabic numerals were not on the fast track. Al-Khowarazmi wrote his book in the ninth century, but the news didn't reach Moslem Spain until the eleventh century. Christian scholars learned the "new" method from the Moslems and brought it to the rest of Europe, where it languished for hundreds of years. Throughout the remainder of the Middle Ages, the Renaissance, the Reconquista of Spain, the conquest and colonization of the New World, the Reformation and the Counter-Reformation and the countless wars, most Europeans, excluding the great Italian banking families, clung to their tried-and-true Roman numerals and counting boards. Why was this so? One reason is that Arabic numerals, which initially require more discipline and work (to learn the arithmetic facts and the algorithms), exceeded the needs of the Europeans. Roman numerals met their needs exactly, no more, no less. When Roman numerals and counting boards failed to meet the needs of the Industrial Revolution, Arabic numerals and algorithms were embraced, at last.

Scholars and scientists also refused to adopt Arabic numerals. They were content with the ancient Babylonian base 60-method, and for another very good reason. Base 60 could deal with decimal fractions; Arabic numerals couldn't (yet). Moving from right to left in base 60, the number in the first (units) place is multiplied by 1, the number in the second place is multiplied by 60, the number in the third place is multiplied by 3,600, and so on. But three thousand years earlier the Mesopotamians had figured out that numbers also could move to the right of the units place. Thus, the first number to the right was multiplied by 1/60, the second number was multiplied by 1/3600, and so on. The powers of 60 could be negative as well as positive.

This is how they would have expressed the powers of 60, had they the use of Arabic numerals and exponents. Powers of 60 could be negative.

60^2	60^1	60^0	60^{-1}	60^{-2}
3600	60	1	1/60	1/3600

It wasn't until the fifteenth century that a Moslem scholar perceived that "10" also could have negative powers.

Powers of 10 can also be negative.

10^2	10^1	10^0	10^{-1}	10^{-2}
100	10	1	1/10	1/100

Again, the new knowledge spread to Europe, and in the seventeenth century the Europeans made one of their few contributions to the Hindu Arabic base 10 place value system, and a splendid one it was: the decimal point. Arabic numerals were ready and waiting for the Industrial Revolution, which arrived one hundred and fifty years later.

The six elements of our modern numbering system

1. Arabic numerals (0, 1, 2, 3, 4, 5, 6, 7, 8, 9)

All of the numbering systems used before the arrival of Arabic numerals were additive. For example, the Romans had a word for "8," *octo*, but they needed 4 symbols to write it, "VIII.' It was necessary to add the values of those symbols to arrive at the intended quantity, 8. In contrast, the quantity represented by the Arabic numeral "8" needs but a single symbol to represent it because the addition already has been done.

2. *Ten*, the fundamental number, is invisible.

Ten is the fundamental number of our base 10, place value system, but we never see it because, unlike the Romans, we have no symbol for it; we never see the fundamental number of any place value system. In base 10, if we start from zero and add single units (0+1=1, 1+1=2, 2+1=3...), we will soon arrive at the fundamental number, *ten*. Just as with an old-fashioned adding machine, the *ten* is represented by a "1" which has been moved to another *place* (the adjoining column to the left), where it now represents 1 *group* of ten "1"s, and a zero, which occupies the first column.

> If, for some reason, we were to convert our place value system from base 10 to base 8, an interesting thing would happen. We would still have a word for 8, *eight*, but we no longer would have use of the numeral. Starting from zero and adding single units, as above, we would arrive at the new fundamental number, *eight*, at which point we would write a "1," which now would mean 1 *group* of eight 1's, in another *place* and append a zero to the right as place holder. If Dana had eight dollars, we would *say*, "Dana has eight dollars," but *write*, "Dana has 10 dollars." Strictly speaking, the *written* term ""base 10" can refer to any base whatsoever. "There are 10 types of people: those who understand binary and those who don't" is a *written* witticism that affords merriment to those who understand that "10" here expresses the *base 2* value of "two."

Continuing on in base 10, if we keep adding single units we soon will accumulate 2 groups of ten units, then 3, 4, and so on until we have 10 groups of 10 units. We can't keep a ten in any column (because *ten*, as our fundamental number, has no symbol), so the new *ten* is rolled over into the next (third) column and represented by a "1," which now means 1 group of 10 x 10 units. Another way to write 10 x 10 is 102, where the "2" is called an exponent (an exponent indicates how many times a number is multiplied by itself). We write it as "102," but we say "10 squared" or "10 to the power of 2."

3. Place value

The *place* in place value refers to the position of the various Arabic numerals (0 - 9) in any number. *Place* determines the final *value* of each number by attaching a hidden multiplier (the fundamental *ten* raised to some power) to that number. To the left of the decimal point, the number in the first *place* is multiplied by 1, the number in the second *place* is multiplied by 10, the number in the third *place* is multiplied by 100, etc. Moving to the right of the decimal point, the number in the first *place* is multiplied by 1/10, the number in the second *place* is multiplied by 1/100, the number in the third *place* is multiplied by 1/1000, etc. The figures below illustrate the meaning of *345.34*.

***The meaning of* 345.34.**

	10^2=100	10^1=10	10^0=1	10^{-1}=1/10	10^{-2}=1/100
	x 3	x 4	x 5	x 3	x 4
345.34 =	300 +	40 +	5 +	3/10 +	4/100

4. Shorthand notation

"345" is shorthand for 300 + 40 + 5, which is shorthand for (3 x 100) + (4 x 10) + (5 x 1), which is shorthand for (3 x 10 x 10) + (4 x 10) + (5 x 10/10). That is a lot of shorthand. The genius of Arabic numerals is that we can manipulate them, using the standard algorithms, without giving thought to the shorthand involved.

> This amazing simplicity of operations is why children can learn to **do** arithmetic without **understanding** it. And the same simplicity is what misleads so many educators. They expect your child to be capable of understanding something that appears to be so simple. But arithmetic is not simple at all, and understanding it is not easy. It is beyond the reach of small children.

5. Zero

The Roman system, where 202 was expressed as CCII, did not require a zero. Our place value system does. Zero has two functions: as a placeholder (in 202, for example, where we could as easily write 2 - 2, or 2   2), and as a working number (5 + 0 = 5). In effect, ***zero* has replaced *ten***.

6. Algorithms

Every numbering system needs an operating system. The Romans used their counting boards; we use algorithms, which are step-by-step procedures for solving certain categories of mathematics problems. Taken together, they form the operating system for Arabic numerals. Each type of arithmetic problem has its own algorithm. For example, here is the algorithm for addition of whole numbers, without carry-over.

Adding 123 and 456

1. Write the problem in vertical form.
2. Add the digits in the "ones" column.
3. Add the digits in the "tens" column.
4. Add the digits in the "hundreds" column.

1	2	3	4
123	123	123	123
+ 456	+ 456	+ 456	+ 456
	9	79	579

The addition algorithm seems such a simple and straightforward way to combine two quantities that there is a temptation to view it as easily discoverable. It is important, however, to realize that this algorithm is the end product of over 5,000 years of human thought. The first agricultural settlements appeared in Mesopotamia between 5,000 and 6,000 BC. The Sumerians, who established the basic features of Mesopotamian civilization, arrived about 3,500 BC. Four thousand years passed before the unsung Hindu genius invented "Arabic" numerals, and it took another 1,200 years or so before the numerals and operating system were refined to their present form. In all the years since the algorithm was published by al-Khowarizmi , no better method of dealing with "123 + 456" has been devised.

As mentioned above, learning Arabic numerals and how to operate them requires discipline and work. The discipline must be imposed and the work must be directed. In other words, it must be taught. We will review the history of how this was done and why it is being done the way it is today, but first let's take a look at the tools Dana will bring to the educational process.

Psychological development of children

A. Infancy

Dana was born into a personal paradise. Call it her Garden of Eden, if you wish. There were three huge differences between her world and ours. First, since her brain could not as yet comprehend the relationship between (apparent) size and distance, she perceived solid objects as being elastic. Her mother's face would grow large, then small, then large again, as if by magic. Second, being physically helpless, Dana did not work. Everything was brought to her; everything was done for her. If things weren't exactly to her liking, she used her inborn instincts to promote change. An involuntary smile or cry made her mother's face grow large. Warmth and comfort and nourishment followed. She maintained eye contact with her mother while nursing, which prolonged the encounter and secured a full meal, and she did those things without volition. Third, she was literally a learning machine and learned without effort. Among her most important powers was an inborn ability to acquire language without formal instruction of any kind. Born with a portion of her brain already hardwired for the task, which area was fully engaged even as her family prattled "meaningless" words to her.

Her horizon expanded as she learned to crawl, then walk. She relentlessly investigated every single aspect of her kingdom and continued to learn at a prodigious rate, effortlessly. By age four, she had acquired at least one complete language, and her language skills could outperform the world's largest computer. She even displayed a glimmering of formal logic when she ran into the house screaming, "Johnny *hurted* me!" because she was applying previous examples of past tense verb formation to current events. And she was absolutely superb at absorbing facts, although she couldn't form a logical connection between chasing a ball into the street and the very grave danger of being hit by a passing car. The power of formal reasoning lay years in the future.

B. Pre-operational stage of development

Dana's magical kingdom began to dissolve as her brain adjusted for the puzzling changes in the size and shape of things. But newer concepts, such as "how much" and "how many," were quite hazy. She would choose orange juice in a tall, narrow glass over an equal volume of orange juice in a short, wide glass because there appeared to be more juice in the tall glass, and she would stay with her choice even when the juice was poured back and forth from one glass to the other as proof that the volume of juice was the same in either glass. She would choose a widely spaced group of 5 jelly beans over a tightly spaced group of 5 jelly beans for the same reason, in spite of having solemnly counted out "5" in each group.

Counting out loud ("one, two, three...") was not much more than a vocabulary building exercise. Although her parents were thrilled that she was "learning her numbers," it had nothing to do with learning math. As her fine motor skills improved and she learned to write symbols for those words (1, 2, 3...), it still had nothing to do with learning math. A loosely spaced "5" was considered to be greater than a tightly spaced "5." Dana was at the "pre-operational" level of development where numbers existed but had no real meaning.

And then one day something happened. Dana could no longer be fooled by the jellybean trick. She had achieved *conservation of number*, whereby the *space* occupied by the jellybeans no longer distorted the *quantity*. It was not something she was taught; it can't be taught. It was a direct consequence of the ongoing development of her brain into a complex, bilateral structure. Dana was slowly moving from her own magical kingdom into the world of reason.

Unfortunately for Dana, there is a tradeoff attached to the growing sophistication of her brain. As her power to reason increases, her ability to learn without effort decreases. Everything previously done so effortlessly was done because of her instinctive tendency to do those things. She does not, however, have an instinctive tendency to learn the addition and subtraction facts. That will require *work*, a concept with which Dana will become familiar. Hopefully, she will learn that work can be rewarding because tears and smiles, while still somewhat effective with Mom and Dad, are proving much less effective with siblings and playmates and teachers.

C. Concrete operational stage of development – attaching numbers to objects

Dana has entered what many psychologists call the *concrete operational* stage of development. Space and motion no longer distort quantity. She conserves number, and so is ready to deal with concrete manipulatives, such as counters or beans, and learn a little math. At this level, *number* is regarded as an attribute of objects, much as color or size or shape or texture. This level of understanding can be built upon and carry her through grade school arithmetic and beyond. The ancient world operated at this level. Armies were paid, grain production for entire empires was analyzed, the Egyptian pyramids, and the Hanging Gardens of Babylon were constructed. But there is a higher level of understanding numbers, a level that views numbers as independent, abstract entities. This level will be beyond Dana until she reaches the final stage of psychological development.

D. Formal operational stage of development – power to reason with formal logic

There was a dramatic change in Dana's mental apparatus when she attained conservation of number and entered the concrete operational stage of development. She won't experience another such change until she is somewhere between 11 and 15 years old, at which time she will enter the final stage of psychological development, called *formal operational* by some psychologists, and become equipped to deal with formal logic and abstract concepts. Just as there was no way to hurry the change from pre-operational to concrete operational, there is no way to hurry the change to formal operational. Her brain will complete its bilateral hardwiring at its own pace. Dana's brain can be nourished and exercised, but it can't be hurried. This is not to say that one day Dana won't be able to employ formal reasoning and the next day she will. Rather, it means that there will come a time when she is ready for an introduction to ideas employing formal logic, but that time is not now.

This final stage of psychological development is marked by the ***potential*** to view numbers in an entirely different way. If this potential is reached, Dana someday will view numbers, not as attributes of concrete objects, but as independent and abstract entities with infinities of forms. It is at this level that the difference between numbers and numerals becomes clear. When *five*, for example, is considered in the abstract; it can be represented by many different forms, an infinity of forms, in fact, much as the abstract concept of beauty can have an infinity of forms. *Five* can be represented symbolically as *5*, *8-3*, *y+2*, *the square root of 25*, *the 3rd prime number*, and so on. Numbers at this level can be viewed as *anti-words* because numbers have only *one meaning*, but many forms, whereas words have *many meanings*, but only one form. (Logicians have developed a synthesis called *Symbolic Logic*, where the "symbols" have one form only and one meaning only, but that way lies madness). Please keep these thoughts in mind when we review the history of New Math and the ruinous effects of introducing abstract concepts to first and second graders.

Now that we know something of the hidden complexity and sophistication of grade school arithmetic and the developmental stage of Dana's brain, it is time to visit the war between those who want Dana to *understand* math and those who just want her to *learn* it.

A brief history of European and American mathematics instruction

A. From the Middle Ages to Sputnik

In Medieval Europe, warfare was the business of the rulers and the burden of the ruled. Many male children judged unsuitable for war duty were encouraged to join the Church, and there they became the clerics who kept the books for the various kingdoms, bishoprics, and fiefdoms. They performed their calculations on counting boards and wrote the answers in Roman numerals. The Industrial Revolution changed that system. Since there were not enough clerics to staff the proliferating counting rooms of Europe, training for mathematics was separated from training for the religious life. Arabic numerals were taught, and the products of this teaching were called clerks (a modification of *cleric*).

The new students tended to be more unruly than their predecessors, and so mind numbing, unending calculations formed the backbone of the curriculum, both as a teaching method and as a disciplinary device. No effort was made to encourage understanding of the material. This system of training students in mathematics was transferred to the New World and remained intact through World War II. But many educators were unhappy with the system and sought a better teaching method, a way to encourage *understanding* of the material, thereby eliminating the need for the constant, boring drills that turned so many students away from math. Though money was scarce, several universities experimented with new teaching methods. These new methods were similar and later were given the collective name of New Math. Experimentation was measured, results were analyzed, and progress, though slow, was steady.

B. From New Math to Newer Math

The successful flight of Russia's *Sputnik* in 1957 changed things forever. The federal government, already worried about potential shortages of qualified engineers and scientists to staff our increasingly technical industrial base, became terrified that we were falling behind the Russians. Purse strings were opened, and harmful effects of the new money soon appeared. The measured pace of research was replaced by a frantic scramble for market share in textbooks and teaching programs. It quickly became a gold rush in which everybody had a license to "improve" the teaching of mathematics. *Set theory* became the grand vehicle that would lead to understanding. Never mind that set theory, until then, had been the domain of professional mathematicians and graduate students. Children who lacked the ability to deal with abstract concepts because of the immaturity of their brain structures were informed of the difference between numbers and numerals, that "5," when written, was a *numeral*, a symbolic representation of the set of all mathematical operations from which an abstract, unwritten *number* "5" could be derived.

The properties of numbers (not numerals) were introduced: associative [(5+6)+7 = 5+(6+7)], commutative (5+6 = 6+5), and distributive [5x(6+7) = 5x6 + 5x7]. These were the tools that would lead to discovery, then understanding. Children were reminded that as soon as they recorded a certain *number* in writing, they were using a *numeral* to do so. There is more. The children were expected, to a great extent, to explore these concepts *on their own!*

Those of you reading the handsome, cyber-bound edition of this booklet on the web may be interested to know of another consequence of Sputnik. A federal bureaucracy called ARPA (Advanced Research Projects Agency) was established to oversee the research activities of increasingly worried military services and increasingly busy defense contractors. Around 1970, a system was devised that would allow various components of ARPA to transfer data via computers in the event of nuclear war. ARPAnet was born. From a union of ARPA and the National Science Foundation came NSFnet. From NSFnet came the Internet, which, as time went by, developed an internal organ called the World Wide Web. And so the beneficial apparatus derived from one consequence of Sputnik enables us to communicate about the harmful effects of another.

New Math was a catastrophe. Teachers were untrained (and untrainable), parents were ignored, school officials were assailed, but only the children were harmed. So great was the damage from New Math that a counterrevolution, called *Back to Basics*, was formed. Back to Basics was gaining momentum when the well funded and well connected (remember the chase for market share) New Math camp counterattacked by calling up painful images of the way math was taught in the bad old days. This is the Space Age, they argued, we must do more than teach future farmers how to plan for next year's crop. New Math had its shortcomings, they admitted, but they had *reformed* the method. In fact, they called it Reform Math. *Reform* was an exquisitely clever choice of words because reform, after all, was what the Back to Basics people wanted. Many parents mistakenly believed that Reform Math and Back to Basics were the same. Parents were pushed out of the loop from the first days of New Math. In earlier days, a child who needed help with math stood a good chance of finding it at home, from a parent or an older sibling. But in a system that even competent and dedicated teachers couldn't comprehend, parents, including those who enjoyed math and were good at it, were helpless because the textbooks seemed to make no sense at all. Parents tilted at windmills as teachers floundered, school district officials shook hands with the textbook dealers, and the various state governors politely yawned and stood above it all. New teaching programs still are introduced with great fanfare, but the programs do not work. *Pattern recognition* is now the rage. It is declared that pattern recognition will lead to *mathpower*. Underlying everything is *exploration* and *group work*, *calculators* and *games*. Many school administrators now pin their hopes on computers, although there probably is not one truly effective math teaching software program available. Talk about a race for market share. (The Math Path computer *program* teaches facts, not mathematics. The Math Path *system* teaches mathematics).

In effect, students are expected to teach themselves. Any way to arrive at a particular answer, any way at all, is fine, for it demonstrates that the student has gained an insight into the underlying process. Beyond that, the worst of the schools don't demand a correct answer. The effort expended, and the improved self image from the praise that effort elicits from the teacher, are quite sufficient. *Mathpower* is sure to follow. Sadly, many teachers never even pose specific arithmetic problems. Instead, they assign groups to discover for themselves, for example, how a small, urban business can turn a profit (so much for the Space Age; at least we've moved off the farm). Group work, of course, provides the perfect cover for shy or lazy children.

There are those who want Dana to *understand* math and those who want her to learn it. For the first group, we can lump together their many programs and call them Newer Math (my term, others use the pejorative "Whole Math," in reference to the equally disastrous "Whole Word" approach to reading, or "Feel Good Math," from the constant praise heaped on flailing students). Parents and some concerned teachers who form the bulk of the second group might call their various programs "Basic Math." The pejorative for this is "rote learning." Newer Math students are encouraged to discover their own algorithms and gain understanding through that process. We have seen that the standard algorithms are the end product of over 5,000 years of human thought on numbers and how to manipulate them, so it's not likely that Dana will duplicate this work in her first few years of grade school. Basic Math students are *taught* the standard algorithms, but there is indifference (duly noted and advertised by the Newer Math crowd) as to the amount of understanding acquired in the process. Here is a summary of the two philosophies:

Newer Math – teaches for *understanding.*
**pejorative: "feel good" math
**algorithms self-taught
**algorithms expected to be understood
**arithmetic operations de-emphasized
**calculator use encouraged
**student performance is hard to measure
**teachers need special skills
**students become comfortable with themselves
**curriculum is wide and shallow

Basic Math – teaches for *performance.*
**pejorative: "rote learning" math
**algorithms taught
**algorithms not expected to be understood
**arithmetic operations emphasized
**calculator use forbidden
**student performance is easy to measure
**teachers need no special skills
**students become comfortable with numbers
**curriculum is narrow and deep

Since there is only so much time and energy available for the acquisition of math skills, common sense tells us that a compromise should be possible between the competing philosophies, but there is no common ground. From the viewpoint of those in the Newer Math group, teaching for understanding and teaching the formal algorithms are mutually exclusive. Teaching a specific algorithm forecloses the exploration process that is the basis of their philosophy. If someone *tells* Dana how to do it, how is she going to *discover* how to do it? Huh?

Consider this. A system of dealing with numbers as sophisticated, though seemingly simple, as our own was developed only once throughout the entire course of human history. Why then, should we believe that the Newer Math crowd is capable of inducing Dana, a pre-adolescent years away from entering the formal operational level of psychological development, to "discover" this system. The answer is that we shouldn't believe it. Dana is capable of learning the algorithms, and of understanding the mechanics involved, but she is not capable of discovering anything of mathematical value on her own. Perhaps someday, in her mature years, Dana will emerge as a giant and invent a better method of dealing with numbers. If that happens, people surely will remark how simple and straightforward her system is, how painfully obvious. What about the Basic Math crowd? They are not opposed, in principle, to children understanding math. But they feel that the present pursuit of understanding reduces the likelihood of their children doing well enough at math to get in and out of a good college and get a good job afterward. Math power may foster Dana's self esteem, but comfort with numbers and expertise with the algorithms will enable her to take her math education as far as her talents will allow. She might even become a graduate student in the mathematics department of a major university. That will be the time to seek true understanding of mathematics, perhaps by investigating set theory or pattern analysis.

The truth about "true" understanding

There is a world of difference between true understanding of math (or even numbers) and reaching a level of competence where things seem to make sense. Bertrand Russell, a very gifted man who sought a true understanding of mathematics, became confused while pursuing the logical basis of 5 plus 3 equals 8. A group of prominent French mathematicians, the Bourbaki Group, produced a paper dealing with the fundamental characteristics of numbers. The entry for "1" ran to over 200 pages.

The plain truth is that Dana probably never will achieve a true understanding of even one segment of math. If she is lucky enough not to become the victim of a "feel good" math program or of math phobia, the math she is learning now and the math she has yet to learn will start to make sense in high school, after her brain has formed the hardwiring necessary to make the logical connections. Well before then, however, she can become expert in performing grade school arithmetic, and even some algebra. By "expert," I mean that she should be able to perform rule-based calculations without consciously thinking of the rules. Using that definition, most of us are expert drivers. We jump in the car, drive to the store, and jump out without having given one thought to the rules of driving. Yet very few of us could be said to truly understand cars. Certainly, when we first learned to drive, no one gave us the keys to a car and told us to drive off and discover the truth about automotive transportation. We were thoroughly grounded in the rules of operating cars, and now we are expert at the operating system and hardly ever think about the rules we so nervously mastered. Our status as expert drivers gives us an independence and freedom without which we would be substantially handicapped in life.

Continuing the automobile analogy, "Why do I have to learn to drive a car?" is a complaint none of us have ever heard a teenager make. The benefits of learning to drive are so obvious and so immediate that failure in that area is not an option. It is, in fact, unthinkable. Whatever actions are necessary to obtain a driver's license will be undertaken. Yet "Why do I have to learn math?" is a constant refrain. The child complains because neither the benefits of success nor the consequences of failure are immediately apparent. But if Dana fails to become expert, first at arithmetic and then at the higher levels of math required for college work, her options in life will be substantially narrowed. Therefore, it is the parents' job to keep in mind that failing to learn math is not an option for Dana. It is unthinkable.

What's a poor Parent to do?

Many parents have discovered that the ruinous effects of the "whole word" reading system can be overcome by a comparatively brief period of tutoring, using any of the various phonics systems that are commercially available. Those parents who have taken advantage of the phonics programs have, in effect, wrested from the school the responsibility for their child's progress in reading. In like manner, the harmful effects of "whole word" math programs can be overcome when parents assume responsibility for their child's progress in basic arithmetic.

Monitoring progress in math is more difficult than in reading. Parents can monitor a child's progress in reading simply by asking her to read for them. Either she has trouble reading or she doesn't. But they can't monitor her progress in Newer Math because they have been cut out of the loop. They don't know what the programs are supposed to do nor how to measure progress.

Tutoring also is more difficult in math because the critical learning period is much longer. Everything in math is built upon, and requires mastery of, things that went before. Memorizing the addition facts requires knowledge of numbers. Executing the addition algorithm requires mastery of the addition facts. The multiplication algorithm requires mastery of multiplication facts, addition facts, and the addition algorithm. Building upon what went before continues through advanced calculus. Mathematics is structured like an upside down pyramid, and basic arithmetic forms the small base. If the base is weak, nothing of mathematical consequence can ever be built upon it.

You must monitor your child's progress in the fundamentals of basic grade school arithmetic. If the school can't or won't teach basic arithmetic, then you must arrange for it to be done elsewhere. You can do it yourself or pay an outsider to do it. Your child must become expert in addition, subtraction, multiplication and division because those areas, together with fractions, form the foundation of higher mathematics. A child who isn't comfortable with the addition facts will experience discomfort with every addition problem. This discomfort will increase as higher levels of arithmetic are built upon an increasingly unstable foundation. Your child is too young to have acquired math phobia or an attitude of "I can't do math," but that time may come if care isn't taken. Start now. Trying to play catch-up by reviewing fundamentals with a recalcitrant sixth grader is nearly impossible.

Your child is at an age where she will cooperate in acquiring math skills. She will proudly display them. But her cooperation will vanish if she develops a distaste for math. She is not old enough to contemplate what contempt for math will mean to her future prospects. Contemplating her future is your job.

Summary

1. Inability to understand math is not a bar to achieving comfort with the mechanical operations and performing them expertly.

2. Arithmetic must be taught. It cannot be discovered.

3. Most adults (and most teenagers) can teach basic arithmetic. The operations are simple and mastery of them is easily measured. Don't complain about the school system to your child. Just do the job that must be done.

4. It is the expectation of success that fuels achievement in mathematics. Schoolchildren can't be fooled by a "feel good" environment. They know they are not really learning math. Accordingly, they learn to anticipate failure instead of success. Learning, really learning, numbers, facts, and operations will foster an expectation of success and help Dana to persevere and succeed on her own in algebra and beyond. You have it in your power to make this happen.

Note from Internet Publisher: Donald L. Potter

October 22, 2008

A couple of years ago, I discovered Jerry Schnell's revolutionary essay, "Why Your Child Can't Understand Math." Jerry sent me his computer program and information on his beliefs concerning how math should be taught. I was very interested because I had pretty well figured out how to teach reading with phonics and had come to the sad realization that students were struggling with a condition aptly called artificially induced whole-word dyslexia by Dr. Samuel Blumenfeld. I discovered how to diagnose and cure it. I was delighted to learn that Mr. Schnell had done similar work in the area of math.

I use his program in my private tutoring and have found it highly effective.

I am publishing his essay with his permission in the earnest hope that home school parents, tutors, teachers, administrators and curriculum developers will consider his crucial insights.

December 24, 2009

Late last year, Mr. Schnell stopped by my school to bring me some of his material. He spent the day at my school discussing the best way to teach math. He was also was very interested in learning about how I cured artificially induced whole-word dyslexia with phonics. We had a very delightful time. We kept in contact for a while after that, but I have unfortunately lost track of him.

I regret that his excellent website is no longer on the Internet. I am glad I kept this file so that the information that was on his web site can be preserved and made available to others. The information is extremely important and needs to be better known.

I hope to publish more of Mr. Schnell's math materials in the future. Eventually I would like to contact Mr. Schnell again to see if we can continue to make his computer math program available through some means. It only runs on IBM. Maybe we can get a programmer to update it.

More information of arithmetic is available on the Math page of my website: www.donpotter.net

Don Potter, Odessa, TX.

Last updated 12/24/09